

AMENDMENTS TO THE SPECIFICATIONS

Please amend paragraph [0005] as indicated:

[0005] A conventional way to determine a localized spectrum is to use the method of the Short Time Fourier Transform (STFT). In STFT, tapered moving windows of the time domain signal are used to compute their Fourier spectra. The general form of this transform is:

$$STFT_{\tau}(f) = \int_{-\infty}^{\infty} f(t)g(t-\tau)e^{-i\omega t}dt$$

where  $f(t)$  is the time-domain function,  $g(t)$  is the window function, and  $e^{-i\omega t}$  is the Fourier kernel. This method has been summarized by Nawab and Qauatieri (see Nawab et al., 1988. "Short-time Fourier transform" in Advanced Topics in Signal Processing, Lim, J., and Oppenheim, Al. Eds.: Prentice Hall Signal Processing Series, 289-337.) and employed in practice by Partyka et al. (see Partyka et al., 1999. "Interpretational aspects of spectral decomposition in reservoir characterization: The Leading Edge, 18, 353-360.) and Marfurt et al. (see Marfurt et al., 2001. "Narrow-band spectral analysis and thin-bed tuning: Geophysics, 66, 1274-1283.) The analysis window function plays an important role in the STFT, where resolution issues are dependent to a large degree on window size. The longer the window size is in time, the better the resolution of the local spectrum in the frequency domain, but the worse the resolution in the time domain. The shorter the window size is, the better the resolution of the local spectrum in the time domain, but the worse the resolution in the frequency domain.

Please amend paragraph [0007] as indicated:

[0007] In one method, for an observed sequence

$$x_n = -\sum_{m=1}^M a_{M,m} x_{n-m} + \varepsilon_n$$

where  $a_{M,m}$  is the AR parameter  $m$  of the  $M^{\text{th}}$  order AR process.  $\varepsilon_n$  represents the noise components. In terms of vectors,

$$\varepsilon_n = X_{M,n}^T A_M$$

where

$$A_M^T = [1, a_{M,1}, \dots, a_{M,M}]$$

$$X_M^T = [x_n, x_{n-1}, \dots, x_{n-M}]$$

and  $T$  denotes the vector transpose. If each side of the above equation is pre-multiplied

by complex conjugate vector  $X_{M,n}^*$  and the expected value taken, then

$$\Phi_M A_M = P_M$$

where

$$\Phi_M = E[X_{M,n}^* X_{M,n}^T] = \begin{bmatrix} \phi_0 & \phi_1 & \dots & \phi_M \\ \phi_1^* & \phi_0 & \dots & \phi_{M-1} \\ \vdots & \vdots & & \vdots \\ \phi_M^* & \phi_{M-1}^* & \dots & \phi_0 \end{bmatrix} = (M+1)(M+1)$$

is the Toeplitz autocorrelation matrix,  $\phi_1 = E(x_j x_{j+t}^*)$  is the autocorrelation function at

lag time  $t$ .  $P_M = [p_m, 0, \dots, 0]^T$  and  $p_M = E[\varepsilon_n \varepsilon_n^*]^T$  is the white noise power spectral

density.

Please amend paragraph [0008] as indicated:

[0008] By definition, the white noise is uncorrelated with all  $x_m$  for  $m < n$ . Thus

$$E[\varepsilon_n x_n^*] = E[\varepsilon_n \varepsilon_n^*]$$


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It is proven that the power spectral density  $S_x(f)$  of the series is related to the input noise power spectral density by

$$S_x(f) = \frac{P_m}{\left| 1 + \sum_{m=1}^M a_{M,m} \exp(-i2\pi f m \Delta t) \right|^2}$$


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Please amend paragraph [0010] as indicated:

[0010] The latest approach to precluding or diminishing the problems due to windowing involves the use of the wavelet analysis. Wavelet analysis is a newly established (since the late 1980s) field in mathematics and signal processing. Like the Fourier transform, the wavelet transform also convolves through a discrete summation or continuous integration the time function (signal) with a kernel function. A method of using wavelets is found in Chakraborty et al. (see Chakraborty et al., 1995, Frequency-time decomposition of seismic data using wavelet-based methods: Geophysics, 60, 1906-1916.) as well as Xia (Xia, L., 1999, Spectral analysis of seismic data using wavelet transform: M.S. Thesis, University of Oklahoma.). In wavelet analysis, a wavelet is used as the kernel function in place of the Fourier kernel. For example, given a function  $f(t)$ , its Fourier transform is:

$$F(f) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad (2)$$

where  $\omega = 2\pi f$  and  $e^{-i\omega t}$  is the Fourier kernel. Its corresponding wavelet transform is:

$$F_{\omega}(\sigma, \tau) = \int_{-\infty}^{\infty} f(t) \overline{\psi_{\sigma, \tau}(t)} dt$$

where  $\overline{\psi}$  is the complex conjugate of wavelet  $\psi$ . Wavelet  $\psi$  is a function that is square-

integrable  $\left( \psi \in L^2(\mathcal{R}) \right)$  having zero mean and localized in both time and frequency.

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Therefore,  $\psi$  represents a family of wavelets that satisfies the conditions:

$$\psi_{\sigma,\tau}(t) = \frac{1}{\sqrt{\sigma}} \psi\left(\frac{t-\tau}{\sigma}\right)$$


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where  $\tau \neq 0$  and  $\sigma \neq 0$ .  $\sigma$  is referred to as the scale of the wavelet and  $\tau$  is referred to

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as the translation parameter. Note that the wavelet is normalized such that  $\|\psi\| = 1$ .

Please amend paragraph [0012] as indicated:

[0012] In MPD, a family of wavelets is defined by the form

$$\psi_{(\sigma,\xi,\tau)}(t) = \frac{1}{\sqrt{\sigma}} \psi\left(\frac{t-\tau}{\sigma}\right) e^{i\xi t}$$


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where  $\tau \neq 0$  and  $\sigma \neq 0$ .  $\sigma$  is referred to as the scale,  $\tau$  as the translation parameter,

and  $\xi$  as the frequency modulation. Each wavelet in the family is called a time-frequency atom. If  $\psi(t)$  is Gaussian, these atoms are called Gabor atoms. As shown by Mallat et al., Gabor atoms provide excellent time-frequency resolution. These atoms have combinations of all possible time and frequency widths and as a result constitute a redundant set. Once atoms are defined, a best match between the signal and these atoms is found by projecting the atoms onto the signal and then computing the maximum. A residue is then computed by subtracting from the original signal the product of the atoms and the cross product of the selected atom and the signal. This decomposition is continued until the energy of the residue falls below some threshold. This method has the ability to obtain a good resolution in both time and frequency for data within an intermediate frequency range like seismic data.